

# The strong coupling, unification, and recent data

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## Abstract

The prediction of the strong coupling assuming (supersymmetric) coupling constant unification is reexamined. We find, using the new electroweak data,  $\alpha_s(M_Z) \approx 0.129 \pm 0.010$ . The implications of the large  $\alpha_s$  value are discussed. The role played by the  $Z$  beauty width is stressed. It is also emphasized that high-energy (but not low-energy) corrections could significantly diminish the prediction. However, unless higher-dimension operators are assumed to be suppressed, at present one cannot place strong constraints on the super-heavy spectrum. Non-leading electroweak threshold corrections are also discussed.

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Assuming the minimal supersymmetric extension of the standard model (MSSM) [1] between the weak and some high scale, one finds [2] that the extrapolated electroweak and strong couplings approximately unify at a scale  $M_G \sim 3 \times 10^{16}$  GeV (the grand unification scale). Alternatively, assuming coupling constant unification, one can use the precisely measured weak angle  $s^2(M_Z)$  and fine-structure constant  $\alpha(M_Z)$  to predict the  $Z$ -pole strong coupling  $\alpha_s(M_Z)$ . Model-dependent corrections are typically of order 10%, i.e., comparable to the experimental uncertainty in  $\alpha_s(M_Z)$ , and need to be included consistently [3]. Below, we update and extend our discussion of the  $\alpha_s(M_Z)$  prediction [3–6]. We find that for the  $t$ -quark pole mass  $m_t^{pole} \gtrsim 160$  GeV, the positive corrections proportional to  $m_t^2$  are sufficiently large that the sum of the (Yukawa, threshold, and operator) model-dependent corrections must cancel or be negative for unification to hold. Ignoring possible high-scale matching corrections,  $\tan \beta \approx 1$  and heavy superpartners are preferred ( $\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$ ). However, large negative high-scale threshold and nonrenormalizable operator (NRO) corrections are possible. The former depend on the details of the grand-unified theory (GUT), while the latter [7] are gravitationally induced and are generic. Below, we review our formalism and discuss our results and their implications. We also comment on non-logarithmic superpartner corrections, implications of the anomalous  $Z \rightarrow b\bar{b}$  width, extended models, and on various aspects of the large QCD coupling. A comprehensive analysis is presented in Ref. [8].

The prediction for  $\alpha_s(M_Z)$  reads<sup>1</sup>

$$\alpha_s(M_Z) = \alpha_s^{OL}(M_Z) + 0.014 + H_{\alpha_s} + \frac{\alpha_s^2(M_Z)}{28\pi} + 3.1 \times 10^{-7} \text{ GeV}^{-2} [(m_t^{pole})^2 - (m_{t_0}^{pole})^2] + \Delta_{\alpha_s}, \quad (1)$$

where

$$\alpha_s^{OL}(M_Z) = \frac{7\alpha(M_Z)}{15s_0^2(M_Z) - 3}, \quad (2)$$

is the lowest order prediction, and<sup>2</sup>

$$s^2(M_Z) = s_0^2(M_Z) - 0.88 \times 10^{-7} \text{ GeV}^{-2} [(m_t^{pole})^2 - (m_{t_0}^{pole})^2], \quad (3)$$

where  $s^2(M_Z)$  is the true ( $\overline{\text{MS}}$ ) weak angle and  $s_0^2$  is the value it would have for  $m_t^{pole} = m_{t_0}^{pole}$ . The 0.014 correction is a (model-independent) two-loop gauge correction and the function  $H_{\alpha_s}$  is a smaller (model-dependent) two-loop Yukawa correction.  $\alpha_s^2/28\pi$  is a finite scheme-dependent term. The model-dependent function  $\Delta_{\alpha_s}$  sums threshold and NRO corrections at low and high scales. Substituting in (1) the ( $\overline{\text{MS}}$ ) input values [9–11]  $\alpha(M_Z) = 1/(127.9 \pm 0.1)$  and

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<sup>1</sup> Hypercharge is properly normalized, i.e.,  $s^2(M_G) = 3/8$ .

<sup>2</sup> We do not explicitly treat smaller logarithmic dependences on  $m_t^{pole}$ . They are included in the uncertainty. The 0.88 factor incorporates higher-order QCD corrections which were not included in [3].

$$s_0^2(M_Z) = 0.2316 \pm 0.0003, \quad (4a)$$

$$m_{t_0}^{pole} = 160_{-12}^{+11} + 13 \ln \frac{m_{h^0}}{M_Z} \quad (4b)$$

( $m_{h^0}$  is SM-like light Higgs boson mass<sup>3</sup>), one has (in the  $\overline{\text{MS}}$  scheme)

$$\alpha_s(M_Z) - \Delta_{\alpha_s} = 0.129 \pm 0.001 + 3.1 \times 10^{-7} \text{ GeV}^{-2} \left[ (m_t^{pole})^2 - (160 \text{ GeV})^2 \right] + H_{\alpha_s}. \quad (5)$$

The higher values of  $m_t^{pole}$  (e.g., compared to [3]) and lower value of the weak angle implied by recent data [11] increase the predicted  $\alpha_s$ . An even higher central  $\alpha_s$  value of 0.130 would be predicted for the value  $m_t^{pole} = 174 \pm 16 \text{ GeV}$  suggested by the CDF  $t$ -quark candidate events [12]. Two-loop Yukawa corrections are negative but are typically negligible. They can be important if the Yukawa couplings of the  $t$  and/or  $b$ -quark,  $h_t$  and  $h_b$ , respectively, are large, i.e., for  $\tan \beta \approx 1$  or  $\tan \beta \gtrsim 50$ . We find [5]

$$-0.003 \lesssim H_{\alpha_s}(h_t, h_b) = H_{\alpha_s}(m_t^{pole}, \tan \beta) \lesssim 0. \quad (6)$$

For  $h_t \sim \max[h_t(m_t)] \sim 1.1$  (and  $h_b \sim 0$ ) one has [3]  $H_{\alpha_s} \sim -0.1 \times \alpha_s^2 \times h_t^2 \sim -0.002$ . In general, one can substitute a one-loop semi-analytic expression for  $h_t^2$  and integrate iteratively [13] (a similar procedure leads to our result for the gauge two-loop correction [3,8]).

The coupling constant unification is shown in detail in Fig. 1 for various values of  $\alpha_s(M_Z) = 0.12 \pm 0.01$  and for  $\Delta_{\alpha_s} = 0$  and  $H_{\alpha_s} \sim -0.0005$ . In the absence of threshold corrections, and for reasonable  $m_t^{pole}$ , coupling unification requires  $\alpha_s(M_Z) \gtrsim 0.127$ . Below, we show that typically  $|\Delta_{\alpha_s}| \lesssim 0.01$ . Thus, we obtain from coupling constant unification, assuming no conspiracies among different model-dependent corrections,  $\alpha_s(M_Z) \gtrsim 0.12$ . This is in a good agreement with  $Z$ -pole extractions of  $\alpha_s$ , but is slightly higher than some extractions based on deep inelastic scattering (DIS) and quarkonium spectra. The prediction is compared with the data in Table 1 (from [14]). The  $\alpha_s$  measurement and the possibility of light gluinos (that correct the  $\alpha_s$  extrapolation between the quarkonium and weak scales by  $\sim 10\%$ ) are further discussed in Ref. [15,16]. We note, in passing, that light colored scalars would correct the  $\alpha_s$  extrapolation negligibly, i.e., a light scalar top would affect the extrapolation of  $\alpha_s$  measured at low-energy to the  $Z$ -pole by less than 1%. Models (in particular, NRO's) can be constructed with large ( $\gtrsim 10 - 20\%$ ) and negative GUT scale contributions to  $\Delta_{\alpha_s}$ . Such models would violate our no-conspiracy assumption, but cannot be excluded. Hence, even if supersymmetry is characterized by experiment and the superpartner contribution to  $\Delta_{\alpha_s}$  (see below) is found to be positive, coupling constant unification will not

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<sup>3</sup> The authors of [11] perform a best fit to all  $W$ ,  $Z$  and neutral current data assuming  $60 \leq m_{h^0} \leq 150 \text{ GeV}$  with a central value  $m_{h^0} = M_Z$  for the SM-like light Higgs boson mass. (Other possible light particle corrections are discussed separately below.) In the (non-supersymmetric) standard model one assumes a larger Higgs mass range  $60 < m_{h^0} < 1000 \text{ GeV}$  with a central value of  $300 \text{ GeV}$ . This leads to the prediction  $m_{t_0}^{pole} = 175 \pm 11_{-19}^{+17} \text{ GeV}$ , where the second uncertainty is from  $m_{h^0}$ .

be completely ruled out even for  $\alpha_s(M_Z) \sim 0.11$ . However, one will be able to sufficiently constrain GUT's only if the superpartner contribution is large and positive (i.e., if NRO's with perturbative coefficients are not sufficient to rectify the prediction).

The situation in the non-supersymmetric extension is quite different since (a) supersymmetry doubles the GUT sector, (b) NRO's are typically suppressed in the non-supersymmetric case by powers of  $(M_G/M_{Planck}) \sim 10^{-5}$ , and (c) the corrections  $\propto \alpha_s^2(M_Z)$  are suppressed by a  $\sim (0.07/0.13)^2$  factor in comparison to the MSSM [3,8]. One can rectify this situation by considering large logarithms and/or certain complicated chain-breaking scenarios with additional particles, i.e., intermediate scales (which, however, could be constructed to be  $\mathcal{O}(10^{16} \text{ GeV})$  [17] or  $\mathcal{O}(1 \text{ TeV})$  [18]). The predictive power of a desert theory is lost in such a case.

Next, we discuss in greater detail the possible model-dependent contributions to the  $\mathcal{O}(10\%)$  correction function

$$\Delta_{\alpha_s} \approx \frac{-19\alpha_s^2}{28\pi} \ln \frac{M_{SUSY}}{M_Z} + \text{GUT threshold corrections} + \text{NRO corrections.} \quad (7)$$

The parameter  $M_{SUSY}$  [3] is a weighted sum of all superpartner and heavy Higgs boson mass logarithms which determines the (leading-logarithm) contribution to  $\Delta_{\alpha_s}$  [3] [ $\Delta_{\alpha_s} \sim -0.003 \ln(M_{SUSY}/M_Z)$ ]. Specifically,

$$M_{SUSY} = \prod_i m_i^{-\frac{5}{38} [4b_1^i - \frac{96}{10}b_2^i + \frac{56}{10}b_3^i]}, \quad (8)$$

where the index  $i$  runs over all superpartner and heavy Higgs particles. We defined the  $\beta$ -function coefficients  $b_j^i \equiv a(S_i)N_i(j)t_i(j)$ , where  $a(S_i) = \frac{1}{3}, \frac{2}{3}, -\frac{11}{3}$  for a particle  $i$  of spin  $S_i = 0, \frac{1}{2}, 1$ , respectively,  $N_i(j)$  is the appropriate multiplicity, and  $t_i(j) = 0, \frac{1}{2}, 2, 3, \frac{3}{5}(\frac{Y}{2})^2$  for a singlet, a particle in the fundamental representation of  $SU(N)$ , an  $SU(2)$  triplet, an  $SU(3)$  octet, and for  $j = 1$  and a particle with hypercharge  $Y$ , respectively. Because of mass non-degeneracies between colored particles (whose masses are sensitive to the gluino mass), the Higgs and Higgsino particles (whose masses are sensitive to  $\mu$ ), and the scalar leptons (whose masses are sensitive to scalar mass boundary condition), and because of the different weights assigned to the different particles,  $M_{SUSY}$  is not simply the geometric mean of the  $m_i$ . In particular, the negative powers in (8) imply that  $M_{SUSY}$  can be (and generally is) much smaller than the actual masses of the superpartners. In Fig. 2 we calculate  $M_{SUSY}$  for more than a thousand arbitrary<sup>4</sup> MSSM's which are consistent with the electroweak symmetry breaking, a neutral lightest supersymmetric particle, and sparticle masses above experimental lower bounds and below  $\sim 2 \text{ TeV}$  (see [19,20]).  $M_{SUSY}$  is proportional to the Higgsino mass parameter  $\mu$  [21] and is indeed lower than the actual superpartner and Higgs boson masses. From Fig. 2 we have the approximate upper bound  $M_{SUSY} \lesssim 250 - 300 \text{ GeV}$  (or the lower bound  $\Delta_{\alpha_s}^{SUSY} \gtrsim -0.003$ ).

As mentioned above,  $H_{\alpha_s}$  is large and negative for  $\tan \beta \approx 1$ . Also,  $M_{SUSY} \propto |\mu| \propto \sqrt{1/[\tan^2 \beta - 1]}$  is maximized in that region of the parameter space ( $M_{SUSY}$  is shown as

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<sup>4</sup>We assume universality of the soft parameters at  $M_G$ . However, similar results for  $M_{SUSY}$  hold in more general scenarios.

a function of  $\tan\beta$  in Fig. 3). The proportionality factor depends on and grows with the superpartner masses. Thus, a heavy spectrum and  $\tan\beta \sim 1$  are slightly preferred. This observation is consistent with  $b-\tau$  Yukawa unification (which we do not require here), which is constrained by the interplay between the large predicted values of  $\alpha_s$  and the Yukawa-unification preference of moderate  $\alpha_s$  values [8]. (The large QCD radiative corrections to  $h_b$  constrain one to regions of the parameter space in which large Yukawa coupling can partially compensate for these corrections<sup>5</sup>.) In that region one has the spectacular constraint on the Higgs boson mass  $m_{h^0} \lesssim 100$  (110) GeV for  $m_t^{pole} \lesssim 160$  (175) GeV at one loop (and a stronger bound applies at two loops) [23,19,8].

It was recently suggested that the  $Z$ -pole couplings should be extracted from the data assuming the full MSSM [24]. This is the case if the model contains some particles (aside from the SM-like Higgs boson) lighter than  $\sim 100 - 150$  GeV. However, assuming the heavy MSSM limit,  $SU(2)$  breaking mixing and other non-leading effects are negligible and our leading-logarithm formula, which is derived using renormalization-group techniques, is an excellent approximation. Otherwise, light particle (non-logarithmic) effects can be accounted for in the same manner used to describe the quadratic  $m_t$  dependence [3,6,8], i.e., by the perturbative expansion<sup>6</sup>

$$s^2(M_Z) = s_0^2(M_Z) + \frac{s_0^2(1 - s_0^2)}{1 - 2s_0^2} [\Delta r_Z^{top} + \Delta r_Z^{SUSY}], \quad (9)$$

where  $\Delta r_Z$  [25] sums (universal) corrections to the  $Z$ -boson mass  $M_Z$ . The leading contribution to  $\Delta r_Z^{top}$  is given in eq. (3) and  $\Delta r_Z^{SUSY}$  has been calculated in Ref. [26]. In fact, it is useful to subtract from  $\Delta r_Z^{SUSY}$  leading logarithms summed by  $M_{SUSY}$  and reserve  $\Delta r_Z^{SUSY}$  to denote only additional contributions of light superpartners. The correction function (7) is modified accordingly,  $\Delta_{\alpha_s} \rightarrow \Delta_{\alpha_s} - 1.16\Delta r_Z^{SUSY}$ . The different contributions to  $\Delta r_Z^{SUSY}$  are correlated in a given model, and their interplay determines its magnitude and overall sign. We find [27] that non-logarithmic corrections typically conspire with the  $m_t^2$  term and increase the  $\alpha_s$  prediction, in some cases, by a few percent. Thus, heavy superpartners are preferred beyond the leading order.

On a similar note, it has been observed that if supersymmetry significantly modifies the  $Z$  hadronic width (so that the  $Z \rightarrow b\bar{b}$  anomaly is accounted for) then  $\alpha_s$  extracted from the  $Z$  line shape is diminished significantly (e.g.,  $0.126 \rightarrow 0.112$ ) [11], and this effect was even promoted as a possible resolution of the discrepancy between low and high-energy extractions of  $\alpha_s$  [16]. Such a scenario would require either light Higgsinos and large Yukawa couplings or a very large  $\tan\beta$  and a light pseudo-scalar Higgs boson [28], i.e.,  $|\mu| \lesssim \mathcal{O}(M_Z)$ . However, a scheme with a small  $\mu$  parameter is not favored in GUT models [20]. From our discussion above it is also clear that a solution involving light Higgsinos (or a light pseudo-scalar) is strongly disfavored by the  $\alpha_s$  prediction:

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<sup>5</sup>Finite superpartner loops [22] modify only the allowed large  $\tan\beta$  region.

<sup>6</sup> One could calculate the corrections to all fitted observables, or risk a minor inconsistency and calculate only (universal) corrections to the input parameter ( $M_Z$  in our case). The latter scheme, which we follow, is sufficient for our current purposes.

1. The extracted  $\alpha_s$  line-shape value would decrease (in agreement, however, with low-energy extractions).
2. The predicted  $\alpha_s$  value would increase due to leading-logarithm [ $\propto -\ln(|\mu|/M_Z)$ ] and possibly non-logarithmic threshold corrections.
3. The central value of the fitted  $m_t^{pole}$  [eq. (4b)] would grow to 163 GeV, further increasing the  $\alpha_s$  prediction by 0.0003.

Thus, the  $Z \rightarrow b\bar{b}$  anomaly, if not resolved, contains strong implications for supersymmetric models and could even rule out the simplest and most attractive unification scenarios.

Lastly, we consider possible high-scale contributions to the correction function. Unlike the MSSM case, in which the particles and their mass range are dictated by the model, the details of the high-scale corrections are ambiguous. In the minimal  $SU(5)$  model [29] negative threshold corrections in (7) due to super-heavy color triplet Higgs supermultiplets are strongly constrained by the non-observation of proton decay [30], and the GUT-scale threshold correction contribution to  $\Delta_{\alpha_s}$  is typically positive. (This observation, however, need not hold in extended models.) Nevertheless, one cannot extract strong constraints on the GUT spectrum. Gravitationally induced operators (suppressed by  $M_G/M_{Planck} \sim 0.001$ ) split the  $M_G$  gauge couplings (in a correlated manner) and correct the  $\alpha_s$  prediction in proportion to their effective strength,  $\eta$ , which is a free parameter and can have either sign. One has<sup>7</sup>  $\Delta_{\alpha_s}^{NRO} \approx 0.005\eta$ . Constraining the NRO corrections to stay perturbative so that the calculation is consistent (higher-order terms are negligible in this case) one has  $|\eta| \lesssim 2$  ( $|\eta| \sim 3$  is an extreme but still acceptable choice). Thus, NRO's with a non-negligible and negative  $\eta$  could smear light and heavy threshold corrections. Unless  $\eta \gtrsim 0$  and/or  $M_{SUSY} \ll M_Z$  (which could also imply positive non-logarithmic corrections), no significant constraints can be placed on the super-heavy spectrum at present. On the other hand, the minimal  $SU(5)$  model (where threshold corrections are strongly constrained) would require NRO's with  $\eta < 0$  if  $\alpha_s(M_Z) \lesssim 0.125$ . (A similar observation was made recently in Ref. [31].) Thus, unification and quantum gravity may be inseparable.

Regarding the unification scale, corrections that increase the unification scale would typically also increase the prediction for  $\alpha_s(M_Z)$  [32], and are thus difficult to construct [in particular, for  $s^2(M_Z) \approx 0.2316$ ]. This is true for contributions to  $\Delta_{\alpha_s}$  as well as for an additional-matter family [ $\alpha_s(M_Z) \rightarrow 0.132$ ] or additional pairs of Higgs doublets [which lead to non-perturbative values of  $\alpha_s(M_Z)$ ]. This is easily understood if we write  $\alpha_s^{OL}$  as a function of the unification scale  $M$  and of  $\alpha(M_Z)$  (see Fig. 4),

$$\alpha_s^{OL}(M_Z) = \frac{8\alpha(M_Z)}{3 - 60\alpha(M_Z)t}, \quad (10)$$

where  $t = (1/2\pi) \ln(M/M_Z)$ . Naively substituting, e.g.,  $M = M_{string} \sim 5 \times 10^{17}$  GeV [33], one has  $\alpha_s(M_Z) > 0.2$ . By carefully adjusting operator and super-heavy threshold correction

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<sup>7</sup>The proportionality factor is calculated here in the  $SU(5)$  theory [3], and its normalization is different by a factor of four than in [3].

contributions to  $\Delta_{\alpha_s}$ , one could increase  $M_G$  by an order of magnitude while maintaining an acceptable prediction for  $\alpha_s$  [4,8]. However, in general, to rectify the string and unification scales (in level-one models) one has to compromise the predictive power of the unification scenario [34] so that the correlation between  $\alpha_s$  and  $t$  is modified.

To conclude, we have shown that typically one expects a large QCD coupling in supersymmetric unified models (and even more so when considering a typical MSSM spectrum). This constitutes an interesting signature and has implications for, e.g., Yukawa unification, corrections to the unification scale, and the overall sign of the correction function  $\Delta_{\alpha_s}$ , and is in possible conflict with low-energy data. However, it does not yet allow a significant constraint on the super-heavy spectrum because of possible gravitational corrections. We also pointed out the interesting role that the  $Z$  hadronic width might play in supersymmetric GUT's, and suggested a simple formula that extends our treatment of  $m_t^{pole}$ -dependent electroweak corrections to the supersymmetric sector.

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## REFERENCES

- [1] For a recent review, see, for example, H. P. Nilles, in *Testing The Standard Model*, ed. M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 633, and references therein.
- [2] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); W. J. Marciano and G. Senjanovic, Phys. Rev. D **25**, 3092 (1982); L. E. Ibanez and G. G. Ross, Phys. Lett. **105B**, 439 (1982); M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982); U. Amaldi et al., Phys. Rev. D **36**, 1385 (1987); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991); J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); U. Amaldi, W. de Boer and H. Furstenuau, *ibid.* **260**, 447 (1991); F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuovo Cimento **104A**, 1817 (1991).
- [3] P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993).
- [4] N. Polonsky, in *Proceedings of the International Workshop on Supersymmetry and Unification of Fundamental Interactions*, Boston, MA, 1993, ed. P. Nath (World Scientific, Singapore, 1993), p. 398.
- [5] P. Langacker and N. Polonsky, Phys. Rev. D **49**, 1454 (1994).
- [6] N. Polonsky, in *New Physics With New Experiments, Proceedings of the XVI Kazimierz Meeting on Elementary Particle Physics*, Kazimierz, Poland, 24-28 May 1993, ed. Z. Ajduk, (World Scientific, Singapore, 1994), p. 322.
- [7] C. T. Hill, Phys. Lett. **135B**, 47 (1984); Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984); P. Langacker and N. Polonsky [3]; L. J. Hall and U. Sarid, Phys. Rev. Lett. **70**, 2673 (1993). Equivalently, non minimal gauge kinetic functions were considered by J. Ellis et al., Phys. Lett. **155B**, 381 (1985); M. Drees, *ibid.* **158**, 409 (1985). Large dimension-six operators (which are not consistent with a perturbative treatment) were considered by A. Datta et al., Hawaii Report No. UH-511-787-94, 1994. Implications for  $b$ - $\tau$  unification are studied in Ref. [5]. For recent discussions, see also Ref. [31].
- [8] N. Polonsky, *Unification and low-energy supersymmetry at one and two-loop orders*, a University of Pennsylvania dissertation, Report No. UPR-0641T, 1994, and references therein.
- [9] We use the  $\overline{\text{MS}}$  scheme of W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); *Erratum ibid.* **68**, 898 (1992); S. Fanchiotti, B. Kniehl and A. Sirlin, Phys. Rev. D **48**, 307 (1993), which decouples  $m_t$  and therefore differs by (small)  $\ln(m_t/M_Z)$  threshold corrections from the alternative scheme of G. Degrandi, S. Fanchiotti and A. Sirlin, Nucl. Phys. **B351**, 49 (1991), which was used in [3].  $\ln m_t/M_Z$  corrections to  $\alpha$  were treated explicitly in [3], leading to the same numerical value of  $\alpha$  used here. However,  $s_{\text{new}}^2(M_Z) - s_{\text{old}}^2(M_Z) \approx -0.0002$ , which is consistent with the 0.0003 residual uncertainty in our  $s_0^2(M_Z)$ .
- [10] The value of (the  $\overline{\text{MS}}$ )  $\alpha(M_Z)^{-1} = 127.9 \pm 0.1$  used here employs the estimate of the hadronic correction  $\Delta\alpha_{\text{had}}$  in F. Jegerlehner, *Renormalizing the Standard Model*, in *Testing The Standard Model*, ed. M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 476. Recently, three new estimates have appeared, which, when translated to the  $\overline{\text{MS}}$  scheme, imply  $\alpha(M_Z)^{-1} = 128.05 \pm 0.10$  (M. L. Swartz, SLAC Report No. PUB-6710),  $\alpha(M_Z)^{-1} = 127.96 \pm 0.06$  (A. D. Martin and D. Zeppenfeld, Wisconsin Report No. MAD-PH-855), and  $\alpha(M_Z)^{-1} = 127.87 \pm 0.10$  (S. Eidelman and F. Jegerlehner, PSI Report No. PR-95-1). The most extreme deviation, suggested by Swartz, would imply a



- central value for  $\alpha_s(M_Z)$  of 0.130 rather than 0.129. The change is mainly due to a lower central value of  $s_0^2(M_Z) = 0.2314 \pm 0.0003$  (for  $m_{t_0}^{pole} = 150$  GeV), implying an increase of 0.0009 in  $\alpha_s(M_Z)$ . This is partially compensated by the direct effect ( $-0.0002$ ) of the lower value of  $\alpha(M_Z)$ .
- [11] J. Erler and P. Langacker, Pennsylvania Report No. UPR-0632T, 1994, and references therein.
  - [12] CDF: F. Abe et al., Phys. Rev. Lett. **73**, 225 (1994).
  - [13] A. Kusenko, unpublished.
  - [14] Recent results, some of which are preliminary, are from Ref. [15]. LEP results are from S. Bethke, talk presented at *Radiative Corrections: Status and Outlook*, Gatlinburg, TN, June 1994; and from Ref. [11]. Lattice results are from A. X. El Khadra et al., Ohio Report No. OHSTPY-HEP-T-93-020; C. T. H. Davies et al., OHSTPY-HEP-T-94-013. Other results are from I. Hinchliffe, in *Review of Particle Properties*, Phys. Rev. D **50**, 1297 (1994).
  - [15] B. R. Webber, Talk given at *The 27<sup>th</sup> International Conference on High Energy Physics*, Glasgow, Scotland, July 20-27, 1994; Cambridge Report No. CAVENDISH-HEP-94-15, 1994, and references therein.
  - [16] The  $\alpha_s$  discrepancy and its possible resolutions were discussed recently by M. Shifman, Minnesota Report No. UMN-TH-1323-94, 1994.
  - [17] L. Lavoura and L. Wolfenstein, Phys. Rev. D **48**, 264 (1993).
  - [18] U. Amaldi et al., Phys. Lett. B **281**, 374 (1992).
  - [19] P. Langacker and N. Polonsky, Phys. Rev. D **50**, 2199 (1994).
  - [20] N. Polonsky and A. Pomarol, Phys. Rev. Lett. **73**, 2292 (1994); Pennsylvania Report No. UPR-0627T, 1994.
  - [21] M. Carena, S. Pokorski and C. E. M. Wagner, Nucl. Phys. **B406**, 59 (1993).
  - [22] L. J. Hall, R. Rattazzi and U. Sarid, LBL Report No. 33997, 1993; M. Carena et al., Nucl. Phys. **B426**, 269 (1994).
  - [23] V. Barger, M. S. Berger, P. Ohmann, and R. J. Phillips, Phys. Lett. B **314**, 351 (1993).
  - [24] A. Faraggi and B. Grinstein, Nucl. Phys. **B422**, 3 (1994); B. Lynn, Stanford Report No. SU-ITP-93-22, 1993; P. Chankowski, Z. Pluciennik and S. Pokorski, Warsaw Report No. IFT-94-19, 1994; R. Barbieri, P. Ciafaloni and A. Strumia, Pisa Report No. IFUP-TH-47-94, 1994. After completion of this work we received a paper by J. Bagger, K. Matchev and D. Pierce, Johns Hopkins Report No. JHU-TIPAC-95001, 1995, where similar issues are discussed and large positive corrections to  $\alpha_s(M_Z)$  are also found.
  - [25] A. Sirlin, Phys. Lett. B **232**, 123 (1989).
  - [26] M. Drees, K. Hagiwara and A. Yamada, Phys. Rev. D **45**, 1725 (1992).
  - [27] J. Erler, P. Langacker and N. Polonsky, work in progress.
  - [28] A. Djouadi et al., Nucl. Phys. **B349**, 48 (1991); M. Boulware and D. Finnel, Phys. Rev. D **44**, 2054 (1991); G. Altarelli, R. Barbieri and F. Caravaglios, Phys. Lett. B **314**, 357 (1993); J. D. Wells, C. Kolda and G. L. Kane, *ibid.* **338**, 219 (1994); M. Carena and C. E. M. Wagner, CERN Report No. TH-7393-94, 1994; J. E. Kim and G. T. Park, Seoul Reports No. SNUTP-94-66; SNUTP-94-95, 1994; D. Garcia, R. A. Jimenez and J. Sola, Barcelona Report No. UAB-FT-343, 1994.
  - [29] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. R. Quinn and S. Weinberg, *ibid.* **33**, 451 (1974); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**,

- 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).
- [30] R. Arnowitt and P. Nath, Phys. Rev. Lett. **69**, 725 (1992); J. Hisano et al., Nucl. Phys. **B402**, 46 (1993); Tohoku Report No. TU-470, 1994.
  - [31] After completion of this work we received the papers by T. Dasgupta, P. Mamales and P. Nath, Northeastern Report, 1995; D. Ring, S. Urano and R. Arnowitt, Texas A&M Rept No. CTP-TAMU-01-95, 1995; where NRO's in the minimal  $SU(5)$  model are studied.
  - [32] We thank Savas Dimopoulos for stressing this point and for suggesting to plot Fig. 4.
  - [33] V. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988).
  - [34] See, for example, I. Antoniadis, J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **272**, 31 (1991); L. E. Ibanez, D. Lust and G. G. Ross, *ibid.* **272**, 251 (1991); A. Faraggi, *ibid.* **302**, 202 (1993); L. E. Ibanez, *ibid.* **318**, 73 (1993); T. Kobayashi, D. Snematsu and Y. Yamagishi, *ibid.* **329**, 27 (1994).

## TABLES

TABLE I. Values of  $\alpha_s(M_Z)$  extracted from different processes (and extrapolated to  $M_Z$  if relevant). The different values are ordered according to the energy scale of the relevant process.

Bjorken sum rules	$0.122^{+0.005}_{-0.009}$
$\tau \rightarrow \text{hadrons}$ (CLEO)	$0.114 \pm 0.003$
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$ep \rightarrow 2 + 1$ jet rate (HERA)	$0.121 \pm 0.015$
$e^+e^-$ event shape (LEP)	$0.123 \pm 0.006$
$Z$ line shape (LEP)	$0.126 \pm 0.005$
Prediction	$0.13 \pm 0.01$

## FIGURES

FIG. 1. MSSM evolution of  $\alpha_{1,2}$  (solid lines) and of  $\alpha_3$  (dashed lines) in the vicinity of the  $\alpha_{1,2}$  unification point (the scale  $M$  is in GeV).  $\alpha_s(M_Z) = 0.110, 0.115, 0.120, 0.125, 0.130$ ;  $m_t^{pole} = 160$  GeV;  $\tan\beta = 4$ ; and  $\Delta_{\alpha_s} = 0$ .

FIG. 2.  $M_{SUSY}$  as a function of the  $\mu$  parameter. The different universal soft parameters and  $\tan\beta$  are picked at random in the allowed parameter space (see text).  $m_t^{pole} = 160$  GeV.  $M_{SUSY} = M_Z$  is denoted for comparison. (All masses are in GeV.)

FIG. 3. Same as in Fig. 2 except a function of  $\tan\beta$ .

FIG. 4. The Z-pole weak angle and strong coupling are predicted as a function of the unification scale  $M$ . A given value of  $s^2(M_Z)$  corresponds to a fixed choice for  $M$ , e.g.,  $s^2(M_Z) = 0.2359$  corresponds to  $M = 10^{16}$  GeV. MSSM  $\beta$ -functions are assumed. Two-loop Yukawa corrections are taken into account assuming  $m_t^{pole} = 160$  GeV and  $\tan\beta = 4$ . ( $\Delta_{\alpha_s} = 0$ .)  $s^2(M_Z) = 0.2316 \pm 0.0003$  and  $\alpha_s(M_Z) = 0.12 \pm 0.01$  are indicated for comparison.

# The strong coupling, unification, and recent data

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## Abstract

The prediction of the strong coupling assuming (supersymmetric) coupling constant unification is reexamined. We find, using the new electroweak data,  $\alpha_s(M_Z) \approx 0.129 \pm 0.010$ . The implications of the large  $\alpha_s$  value are discussed. The role played by the  $Z$  beauty width is stressed. It is also emphasized that high-energy (but not low-energy) corrections could significantly diminish the prediction. However, unless higher-dimension operators are assumed to be suppressed, at present one cannot place strong constraints on the super-heavy spectrum. Non-leading electroweak threshold corrections are also discussed.

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Assuming the minimal supersymmetric extension of the standard model (MSSM) [1] between the weak and some high scale, one finds [2] that the extrapolated electroweak and strong couplings approximately unify at a scale  $M_G \sim 3 \times 10^{16}$  GeV (the grand unification scale). Alternatively, assuming coupling constant unification, one can use the precisely measured weak angle  $s^2(M_Z)$  and fine-structure constant  $\alpha(M_Z)$  to predict the  $Z$ -pole strong coupling  $\alpha_s(M_Z)$ . Model-dependent corrections are typically of order 10%, i.e., comparable to the experimental uncertainty in  $\alpha_s(M_Z)$ , and need to be included consistently [3]. Below, we update and extend our discussion of the  $\alpha_s(M_Z)$  prediction [3–6]. We find that for the  $t$ -quark pole mass  $m_t^{pole} \gtrsim 160$  GeV, the positive corrections proportional to  $m_t^2$  are sufficiently large that the sum of the (Yukawa, threshold, and operator) model-dependent corrections must cancel or be negative for unification to hold. Ignoring possible high-scale matching corrections,  $\tan \beta \approx 1$  and heavy superpartners are preferred ( $\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$ ). However, large negative high-scale threshold and nonrenormalizable operator (NRO) corrections are possible. The former depend on the details of the grand-unified theory (GUT), while the latter [7] are gravitationally induced and are generic. Below, we review our formalism and discuss our results and their implications. We also comment on non-logarithmic superpartner corrections, implications of the anomalous  $Z \rightarrow b\bar{b}$  width, extended models, and on various aspects of the large QCD coupling. A comprehensive analysis is presented in Ref. [8].

The prediction for  $\alpha_s(M_Z)$  reads<sup>1</sup>

$$\alpha_s(M_Z) = \alpha_s^{\text{OL}}(M_Z) + 0.014 + H_{\alpha_s} + \frac{\alpha_s^2(M_Z)}{28\pi} + 3.1 \times 10^{-7} \text{ GeV}^{-2} \left[ (m_t^{pole})^2 - (m_{t_0}^{pole})^2 \right] + \Delta_{\alpha_s}, \quad (1)$$

where

$$\alpha_s^{\text{OL}}(M_Z) = \frac{7\alpha(M_Z)}{15s_0^2(M_Z) - 3}, \quad (2)$$

is the lowest order prediction, and<sup>2</sup>

$$s^2(M_Z) = s_0^2(M_Z) - 0.88 \times 10^{-7} \text{ GeV}^{-2} \left[ (m_t^{pole})^2 - (m_{t_0}^{pole})^2 \right], \quad (3)$$

where  $s^2(M_Z)$  is the true ( $\overline{\text{MS}}$ ) weak angle and  $s_0^2$  is the value it would have for  $m_t^{pole} = m_{t_0}^{pole}$ . The 0.014 correction is a (model-independent) two-loop gauge correction and the function  $H_{\alpha_s}$  is a smaller (model-dependent) two-loop Yukawa correction.  $\alpha_s^2/28\pi$  is a finite scheme-dependent term. The model-dependent function  $\Delta_{\alpha_s}$  sums threshold and NRO corrections at low and high scales. Substituting in (1) the ( $\overline{\text{MS}}$ ) input values [9–11]  $\alpha(M_Z) = 1/(127.9 \pm 0.1)$  and

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<sup>1</sup>Hypercharge is properly normalized, i.e.,  $s^2(M_G) = 3/8$ .

<sup>2</sup>We do not explicitly treat smaller logarithmic dependences on  $m_t^{pole}$ . They are included in the uncertainty. The 0.88 factor incorporates higher-order QCD corrections which were not included in [3].

$$s_0^2(M_Z) = 0.2316 \pm 0.0003, \quad (4a)$$

$$m_{t_0}^{pole} = 160_{-12}^{+11} + 13 \ln \frac{m_{h^0}}{M_Z} \quad (4b)$$

( $m_{h^0}$  is SM-like light Higgs boson mass<sup>3</sup>), one has (in the  $\overline{\text{MS}}$  scheme)

$$\alpha_s(M_Z) - \Delta_{\alpha_s} = 0.129 \pm 0.001 + 3.1 \times 10^{-7} \text{ GeV}^{-2} [(m_t^{pole})^2 - (160 \text{ GeV})^2] + H_{\alpha_s}. \quad (5)$$

The higher values of  $m_t^{pole}$  (e.g., compared to [3]) and lower value of the weak angle implied by recent data [11] increase the predicted  $\alpha_s$ . An even higher central  $\alpha_s$  value of 0.130 would be predicted for the value  $m_t^{pole} = 174 \pm 16 \text{ GeV}$  suggested by the CDF  $t$ -quark candidate events [12]. Two-loop Yukawa corrections are negative but are typically negligible. They can be important if the Yukawa couplings of the  $t$  and/or  $b$ -quark,  $h_t$  and  $h_b$ , respectively, are large, i.e., for  $\tan \beta \approx 1$  or  $\tan \beta \gtrsim 50$ . We find [5]

$$-0.003 \lesssim H_{\alpha_s}(h_t, h_b) = H_{\alpha_s}(m_t^{pole}, \tan \beta) \lesssim 0. \quad (6)$$

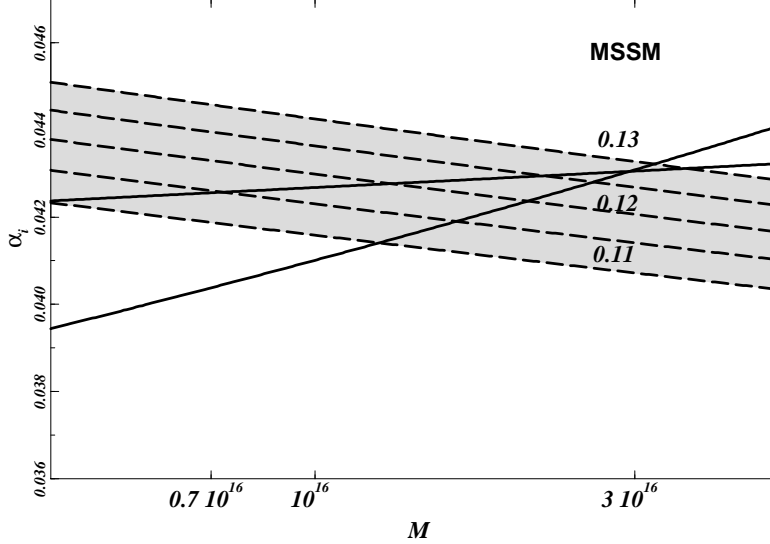
For  $h_t \sim \max[h_t(m_t)] \sim 1.1$  (and  $h_b \sim 0$ ) one has [3]  $H_{\alpha_s} \sim -0.1 \times \alpha_s^2 \times h_t^2 \sim -0.002$ . In general, one can substitute a one-loop semi-analytic expression for  $h_t^2$  and integrate iteratively [13] (a similar procedure leads to our result for the gauge two-loop correction [3,8]).

The coupling constant unification is shown in detail in Fig. 1 for various values of  $\alpha_s(M_Z) = 0.12 \pm 0.01$  and for  $\Delta_{\alpha_s} = 0$  and  $H_{\alpha_s} \sim -0.0005$ . In the absence of threshold corrections, and for reasonable  $m_t^{pole}$ , coupling unification requires  $\alpha_s(M_Z) \gtrsim 0.127$ . Below, we show that typically  $|\Delta_{\alpha_s}| \lesssim 0.01$ . Thus, we obtain from coupling constant unification, assuming no conspiracies among different model-dependent corrections,  $\alpha_s(M_Z) \gtrsim 0.12$ . This is in a good agreement with  $Z$ -pole extractions of  $\alpha_s$ , but is slightly higher than some extractions based on deep inelastic scattering (DIS) and quarkonium spectra. The prediction is compared with the data in Table 1 (from [14]). The  $\alpha_s$  measurement and the possibility of light gluinos (that correct the  $\alpha_s$  extrapolation between the quarkonium and weak scales by  $\sim 10\%$ ) are further discussed in Ref. [15,16]. We note, in passing, that light colored scalars would correct the  $\alpha_s$  extrapolation negligibly, i.e., a light scalar top would affect the extrapolation of  $\alpha_s$  measured at low-energy to the  $Z$ -pole by less than 1%. Models (in particular, NRO's) can be constructed with large ( $\gtrsim 10 - 20\%$ ) and negative GUT scale contributions to  $\Delta_{\alpha_s}$ . Such models would violate our no-conspiracy assumption, but cannot be excluded. Hence, even if supersymmetry is characterized by experiment and the superpartner contribution to  $\Delta_{\alpha_s}$  (see below) is found to be positive, coupling constant unification will not be completely ruled out even for  $\alpha_s(M_Z) \sim 0.11$ . However, one will be able to sufficiently constrain GUT's only if the superpartner contribution is large and positive (i.e., if NRO's with perturbative coefficients are not sufficient to rectify the prediction).

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<sup>3</sup>The authors of [11] perform a best fit to all  $W$ ,  $Z$  and neutral current data assuming  $60 \leq m_{h^0} \leq 150 \text{ GeV}$  with a central value  $m_{h^0} = M_Z$  for the SM-like light Higgs boson mass. (Other possible light particle corrections are discussed separately below.) In the (non-supersymmetric) standard model one assumes a larger Higgs mass range  $60 < m_{h^0} < 1000 \text{ GeV}$  with a central value of 300 GeV. This leads to the prediction  $m_{t_0}^{pole} = 175 \pm 11_{-19}^{+17} \text{ GeV}$ , where the second uncertainty is from  $m_{h^0}$ .

FIG. 1. MSSM evolution of  $\alpha_{1,2}$  (solid lines) and of  $\alpha_3$  (dashed lines) in the vicinity of the  $\alpha_{1,2}$  unification point (the scale  $M$  is in GeV).  $\alpha_s(M_Z) = 0.110, 0.115, 0.120, 0.125, 0.130$ ;  $m_t^{pole} = 160$  GeV;  $\tan\beta = 4$ ; and  $\Delta_{\alpha_s} = 0$ .



The situation in the non-supersymmetric extension is quite different since (a) supersymmetry doubles the GUT sector, (b) NRO's are typically suppressed in the non-supersymmetric case by powers of  $(M_G/M_{Planck}) \sim 10^{-5}$ , and (c) the corrections  $\propto \alpha_s^2(M_Z)$  are suppressed by a  $\sim (0.07/0.13)^2$  factor in comparison to the MSSM [3,8]. One can rectify this situation by considering large logarithms and/or certain complicated chain-breaking scenarios with additional particles, i.e., intermediate scales (which, however, could be constructed to be  $\mathcal{O}(10^{16})$  GeV [17] or  $\mathcal{O}(1)$  TeV [18]). The predictive power of a desert theory is lost in such a case.

Next, we discuss in greater detail the possible model-dependent contributions to the  $\mathcal{O}(10\%)$  correction function

$$\Delta_{\alpha_s} \approx \frac{-19\alpha_s^2}{28\pi} \ln \frac{M_{SUSY}}{M_Z} + \text{GUT threshold corrections} + \text{NRO corrections}. \quad (7)$$

The parameter  $M_{SUSY}$  [3] is a weighted sum of all superpartner and heavy Higgs boson mass logarithms which determines the (leading-logarithm) contribution to  $\Delta_{\alpha_s}$  [3] [ $\Delta_{\alpha_s} \sim -0.003 \ln(M_{SUSY}/M_Z)$ ]. Specifically,

$$M_{SUSY} = \prod_i m_i^{-\frac{5}{38} [4b_1^i - \frac{96}{10}b_2^i + \frac{56}{10}b_3^i]}, \quad (8)$$

where the index  $i$  runs over all superpartner and heavy Higgs particles. We defined the  $\beta$ -function coefficients  $b_j^i \equiv a(S_i)N_i(j)t_i(j)$ , where  $a(S_i) = \frac{1}{3}, \frac{2}{3}, -\frac{11}{3}$  for a particle  $i$  of spin  $S_i = 0, \frac{1}{2}, 1$ , respectively,  $N_i(j)$  is the appropriate multiplicity, and  $t_i(j) = 0, \frac{1}{2}, 2, 3, \frac{3}{5}(\frac{Y}{2})^2$  for a singlet, a particle in the fundamental representation of  $SU(N)$ , an  $SU(2)$  triplet, an  $SU(3)$  octet, and for  $j = 1$  and a particle with hypercharge  $Y$ , respectively. Because of mass non-degeneracies between colored particles (whose masses are sensitive to the gluino mass), the Higgs and Higgsino particles (whose masses are sensitive to  $\mu$ ), and the scalar leptons



TABLE I. Values of  $\alpha_s(M_Z)$  extracted from different processes (and extrapolated to  $M_Z$  if relevant). The different values are ordered according to the energy scale of the relevant process.

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$Z$ line shape (LEP)	$0.126 \pm 0.005$
Prediction	$0.13 \pm 0.01$

(whose masses are sensitive to scalar mass boundary condition), and because of the different weights assigned to the different particles,  $M_{SUSY}$  is not simply the geometric mean of the  $m_i$ . In particular, the negative powers in (8) imply that  $M_{SUSY}$  can be (and generally is) much smaller than the actual masses of the superpartners. In Fig. 2 we calculate  $M_{SUSY}$  for more than a thousand arbitrary<sup>4</sup> MSSM's which are consistent with the electroweak symmetry breaking, a neutral lightest supersymmetric particle, and sparticle masses above experimental lower bounds and below  $\sim 2$  TeV (see [19,20]).  $M_{SUSY}$  is proportional to the Higgsino mass parameter  $\mu$  [21] and is indeed lower than the actual superpartner and Higgs boson masses. From Fig. 2 we have the approximate upper bound  $M_{SUSY} \lesssim 250 - 300$  GeV (or the lower bound  $\Delta_{\alpha_s}^{SUSY} \gtrsim -0.003$ ).

As mentioned above,  $H_{\alpha_s}$  is large and negative for  $\tan\beta \approx 1$ . Also,  $M_{SUSY} \propto |\mu| \propto \sqrt{1/[\tan^2\beta - 1]}$  is maximized in that region of the parameter space ( $M_{SUSY}$  is shown as a function of  $\tan\beta$  in Fig. 3). The proportionality factor depends on and grows with the superpartner masses. Thus, a heavy spectrum and  $\tan\beta \sim 1$  are slightly preferred. This observation is consistent with  $b-\tau$  Yukawa unification (which we do not require here), which is constrained by the interplay between the large predicted values of  $\alpha_s$  and the Yukawa-unification preference of moderate  $\alpha_s$  values [8]. (The large QCD radiative corrections to  $h_b$  constrain one to regions of the parameter space in which large Yukawa coupling can partially compensate for these corrections<sup>5</sup>.) In that region one has the spectacular constraint on the Higgs boson mass  $m_{h^0} \lesssim 100$  (110) GeV for  $m_t^{pole} \lesssim 160$  (175) GeV at one loop (and a stronger bound applies at two loops) [23,19,8].

It was recently suggested that the  $Z$ -pole couplings should be extracted from the data assuming the full MSSM [24]. This is the case if the model contains some particles (aside from the SM-like Higgs boson) lighter than  $\sim 100 - 150$  GeV. However, assuming the heavy

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<sup>4</sup>We assume universality of the soft parameters at  $M_G$ . However, similar results for  $M_{SUSY}$  hold in more general scenarios.

<sup>5</sup>Finite superpartner loops [22] modify only the allowed large  $\tan\beta$  region.

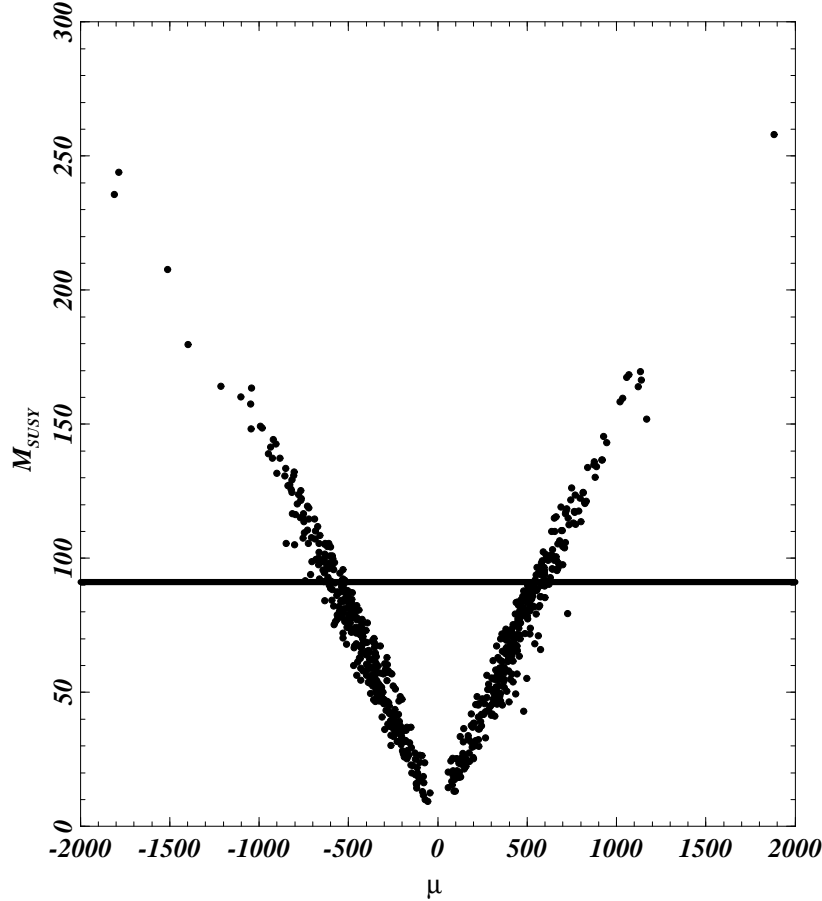


FIG. 2.  $M_{SUSY}$  as a function of the  $\mu$  parameter. The different universal soft parameters and  $\tan\beta$  are picked at random in the allowed parameter space (see text).  $m_t^{pole} = 160$  GeV.  $M_{SUSY} = M_Z$  is denoted for comparison. (All masses are in GeV.)

MSSM limit,  $SU(2)$  breaking mixing and other non-leading effects are negligible and our leading-logarithm formula, which is derived using renormalization-group techniques, is an excellent approximation. Otherwise, light particle (non-logarithmic) effects can be accounted for in the same manner used to describe the quadratic  $m_t$  dependence [3,6,8], i.e., by the perturbative expansion<sup>6</sup>

$$s^2(M_Z) = s_0^2(M_Z) + \frac{s_0^2(1 - s_0^2)}{1 - 2s_0^2} [\Delta r_Z^{top} + \Delta r_Z^{SUSY}], \quad (9)$$

where  $\Delta r_Z$  [25] sums (universal) corrections to the  $Z$ -boson mass  $M_Z$ . The leading contribution to  $\Delta r_Z^{top}$  is given in eq. (3) and  $\Delta r_Z^{SUSY}$  has been calculated in Ref. [26]. In fact, it is useful to subtract from  $\Delta r_Z^{SUSY}$  leading logarithms summed by  $M_{SUSY}$  and reserve  $\Delta r_Z^{SUSY}$  to denote only additional contributions of light superpartners. The correction function (7)

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<sup>6</sup>One could calculate the corrections to all fitted observables, or risk a minor inconsistency and calculate only (universal) corrections to the input parameter ( $M_Z$  in our case). The latter scheme, which we follow, is sufficient for our current purposes.

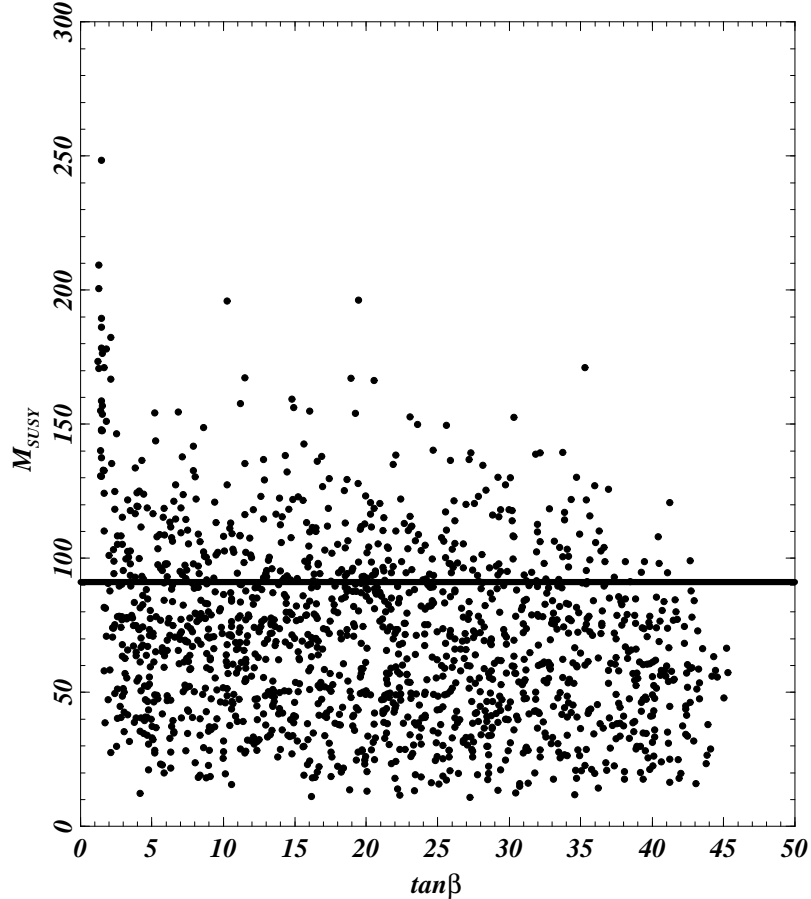


FIG. 3. Same as in Fig. 2 except a function of  $\tan \beta$ .

is modified accordingly,  $\Delta_{\alpha_s} \rightarrow \Delta_{\alpha_s} - 1.16 \Delta r_Z^{\text{SUSY}}$ . The different contributions to  $\Delta r_Z^{\text{SUSY}}$  are correlated in a given model, and their interplay determines its magnitude and overall sign. We find [27] that non-logarithmic corrections typically conspire with the  $m_t^2$  term and increase the  $\alpha_s$  prediction, in some cases, by a few percent. Thus, heavy superpartners are preferred beyond the leading order.

On a similar note, it has been observed that if supersymmetry significantly modifies the  $Z$  hadronic width (so that the  $Z \rightarrow b\bar{b}$  anomaly is accounted for) then  $\alpha_s$  extracted from the  $Z$  line shape is diminished significantly (e.g.,  $0.126 \rightarrow 0.112$ ) [11], and this effect was even promoted as a possible resolution of the discrepancy between low and high-energy extractions of  $\alpha_s$  [16]. Such a scenario would require either light Higgsinos and large Yukawa couplings or a very large  $\tan \beta$  and a light pseudo-scalar Higgs boson [28], i.e.,  $|\mu| \lesssim \mathcal{O}(M_Z)$ . However, a scheme with a small  $\mu$  parameter is not favored in GUT models [20]. From our discussion above it is also clear that a solution involving light Higgsinos (or a light pseudo-scalar) is strongly disfavored by the  $\alpha_s$  prediction:

1. The extracted  $\alpha_s$  line-shape value would decrease (in agreement, however, with low-energy extractions).
2. The predicted  $\alpha_s$  value would increase due to leading-logarithm [ $\propto -\ln(|\mu|/M_Z)$ ] and possibly non-logarithmic threshold corrections.

3. The central value of the fitted  $m_t^{pole}$  [eq. (4b)] would grow to 163 GeV, further increasing the  $\alpha_s$  prediction by 0.0003.

Thus, the  $Z \rightarrow b\bar{b}$  anomaly, if not resolved, contains strong implications for supersymmetric models and could even rule out the simplest and most attractive unification scenarios.

Lastly, we consider possible high-scale contributions to the correction function. Unlike the MSSM case, in which the particles and their mass range are dictated by the model, the details of the high-scale corrections are ambiguous. In the minimal  $SU(5)$  model [29] negative threshold corrections in (7) due to super-heavy color triplet Higgs supermultiplets are strongly constrained by the non-observation of proton decay [30], and the GUT-scale threshold correction contribution to  $\Delta_{\alpha_s}$  is typically positive. (This observation, however, need not hold in extended models.) Nevertheless, one cannot extract strong constraints on the GUT spectrum. Gravitationally induced operators (suppressed by  $M_G/M_{Planck} \sim 0.001$ ) split the  $M_G$  gauge couplings (in a correlated manner) and correct the  $\alpha_s$  prediction in proportion to their effective strength,  $\eta$ , which is a free parameter and can have either sign. One has<sup>7</sup>  $\Delta_{\alpha_s}^{NRO} \approx 0.005\eta$ . Constraining the NRO corrections to stay perturbative so that the calculation is consistent (higher-order terms are negligible in this case) one has  $|\eta| \lesssim 2$  ( $|\eta| \sim 3$  is an extreme but still acceptable choice). Thus, NRO's with a non-negligible and negative  $\eta$  could smear light and heavy threshold corrections. Unless  $\eta \gtrsim 0$  and/or  $M_{SUSY} \ll M_Z$  (which could also imply positive non-logarithmic corrections), no significant constraints can be placed on the super-heavy spectrum at present. On the other hand, the minimal  $SU(5)$  model (where threshold corrections are strongly constrained) would require NRO's with  $\eta < 0$  if  $\alpha_s(M_Z) \lesssim 0.125$ . (A similar observation was made recently in Ref. [31].) Thus, unification and quantum gravity may be inseparable.

Regarding the unification scale, corrections that increase the unification scale would typically also increase the prediction for  $\alpha_s(M_Z)$  [32], and are thus difficult to construct [in particular, for  $s^2(M_Z) \approx 0.2316$ ]. This is true for contributions to  $\Delta_{\alpha_s}$  as well as for an additional matter family [ $\alpha_s(M_Z) \rightarrow 0.132$ ] or additional pairs of Higgs doublets [which lead to non-perturbative values of  $\alpha_s(M_Z)$ ]. This is easily understood if we write  $\alpha_s^{OL}$  as a function of the unification scale  $M$  and of  $\alpha(M_Z)$  (see Fig. 4),

$$\alpha_s^{OL}(M_Z) = \frac{8\alpha(M_Z)}{3 - 60\alpha(M_Z)t}, \quad (10)$$

where  $t = (1/2\pi)\ln(M/M_Z)$ . Naively substituting, e.g.,  $M = M_{string} \sim 5 \times 10^{17}$  GeV [33], one has  $\alpha_s(M_Z) > 0.2$ . By carefully adjusting operator and super-heavy threshold correction contributions to  $\Delta_{\alpha_s}$ , one could increase  $M_G$  by an order of magnitude while maintaining an acceptable prediction for  $\alpha_s$  [4,8]. However, in general, to rectify the string and unification scales (in level-one models) one has to compromise the predictive power of the unification scenario [34] so that the correlation between  $\alpha_s$  and  $t$  is modified.

To conclude, we have shown that typically one expects a large QCD coupling in supersymmetric unified models (and even more so when considering a typical MSSM spectrum).

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<sup>7</sup>The proportionality factor is calculated here in the  $SU(5)$  theory [3], and its normalization is different by a factor of four than in [3].

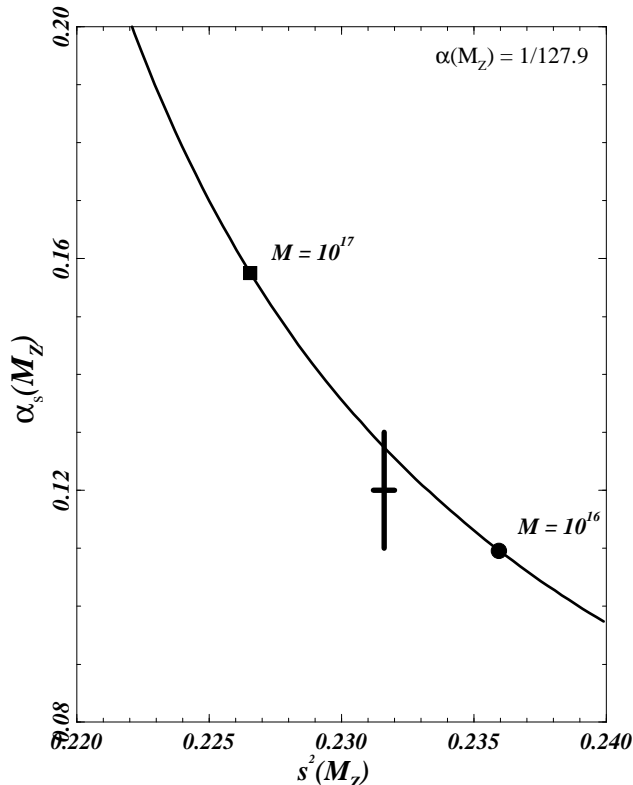


FIG. 4. The Z-pole weak angle and strong coupling are predicted as a function of the unification scale  $M$ . A given value of  $s^2(M_Z)$  corresponds to a fixed choice for  $M$ , e.g.,  $s^2(M_Z) = 0.2359$  corresponds to  $M = 10^{16}$  GeV. MSSM  $\beta$ -functions are assumed. Two-loop Yukawa corrections are taken into account assuming  $m_t^{pole} = 160$  GeV and  $\tan \beta = 4$ . ( $\Delta_{\alpha_s} = 0$ .)  $s^2(M_Z) = 0.2316 \pm 0.0003$  and  $\alpha_s(M_Z) = 0.12 \pm 0.01$  are indicated for comparison.

This constitutes an interesting signature and has implications for, e.g., Yukawa unification, corrections to the unification scale, and the overall sign of the correction function  $\Delta_{\alpha_s}$ , and is in possible conflict with low-energy data. However, it does not yet allow a significant constraint on the super-heavy spectrum because of possible gravitational corrections. We also pointed out the interesting role that the  $Z$  hadronic width might play in supersymmetric GUT's, and suggested a simple formula that extends our treatment of  $m_t^{pole}$ -dependent electroweak corrections to the supersymmetric sector.

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## REFERENCES

- [1] For a recent review, see, for example, H. P. Nilles, in *Testing The Standard Model*, ed. M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 633, and references therein.
- [2] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); W. J. Marciano and G. Senjanovic, Phys. Rev. D **25**, 3092 (1982); L. E. Ibanez and G. G. Ross, Phys. Lett. **105B**, 439 (1982); M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982); U. Amaldi et al., Phys. Rev. D **36**, 1385 (1987); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991); J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); U. Amaldi, W. de Boer and H. Furstenuau, *ibid.* **260**, 447 (1991); F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuovo Cimento **104A**, 1817 (1991).
- [3] P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993).
- [4] N. Polonsky, in *Proceedings of the International Workshop on Supersymmetry and Unification of Fundamental Interactions*, Boston, MA, 1993, ed. P. Nath (World Scientific, Singapore, 1993), p. 398.
- [5] P. Langacker and N. Polonsky, Phys. Rev. D **49**, 1454 (1994).
- [6] N. Polonsky, in *New Physics With New Experiments, Proceedings of the XVI Kazimierz Meeting on Elementary Particle Physics*, Kazimierz, Poland, 24-28 May 1993, ed. Z. Ajduk, (World Scientific, Singapore, 1994), p. 322.
- [7] C. T. Hill, Phys. Lett. **135B**, 47 (1984); Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984); P. Langacker and N. Polonsky [3]; L. J. Hall and U. Sarid, Phys. Rev. Lett. **70**, 2673 (1993). Equivalently, non minimal gauge kinetic functions were considered by J. Ellis et al., Phys. Lett. **155B**, 381 (1985); M. Drees, *ibid.* **158**, 409 (1985). Large dimension-six operators (which are not consistent with a perturbative treatment) were considered by A. Datta et al., Hawaii Report No. UH-511-787-94, 1994. Implications for  $b$ - $\tau$  unification are studied in Ref. [5]. For recent discussions, see also Ref. [31].
- [8] N. Polonsky, *Unification and low-energy supersymmetry at one and two-loop orders*, a University of Pennsylvania dissertation, Report No. UPR-0641T, 1994, and references therein.
- [9] We use the  $\overline{\text{MS}}$  scheme of W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); *Erratum ibid.* **68**, 898 (1992); S. Fanchiotti, B. Kniehl and A. Sirlin, Phys. Rev. D **48**, 307 (1993), which decouples  $m_t$  and therefore differs by (small)  $\ln(m_t/M_Z)$  threshold corrections from the alternative scheme of G. Degrassi, S. Fanchiotti and A. Sirlin, Nucl. Phys. **B351**, 49 (1991), which was used in [3].  $\ln m_t/M_Z$  corrections to  $\alpha$  were treated explicitly in [3], leading to the same numerical value of  $\alpha$  used here. However,  $s_{\text{new}}^2(M_Z) - s_{\text{old}}^2(M_Z) \approx -0.0002$ , which is consistent with the 0.0003 residual uncertainty in our  $s_0^2(M_Z)$ .
- [10] The value of (the  $\overline{\text{MS}}$ )  $\alpha(M_Z)^{-1} = 127.9 \pm 0.1$  used here employs the estimate of the hadronic correction  $\Delta\alpha_{\text{had}}$  in F. Jegerlehner, *Renormalizing the Standard Model*, in *Testing The Standard Model*, ed. M. Cvetič and P. Langacker (World Scientific, Singapore, 1991), p. 476. Recently, three new estimates have appeared, which, when translated to the  $\overline{\text{MS}}$  scheme, imply  $\alpha(M_Z)^{-1} = 128.05 \pm 0.10$  (M. L. Swartz, SLAC Report No. PUB-6710),  $\alpha(M_Z)^{-1} = 127.96 \pm 0.06$  (A. D. Martin and D. Zeppenfeld, Wisconsin Report No. MAD-PH-855), and  $\alpha(M_Z)^{-1} = 127.87 \pm 0.10$  (S. Eidelman and F. Jegerlehner, PSI Report No. PR-95-1). The most extreme deviation, suggested by Swartz, would imply a

- central value for  $\alpha_s(M_Z)$  of 0.130 rather than 0.129. The change is mainly due to a lower central value of  $s_0^2(M_Z) = 0.2314 \pm 0.0003$  (for  $m_{t_0}^{pole} = 150$  GeV), implying an increase of 0.0009 in  $\alpha_s(M_Z)$ . This is partially compensated by the direct effect ( $-0.0002$ ) of the lower value of  $\alpha(M_Z)$ .
- [11] J. Erler and P. Langacker, Pennsylvania Report No. UPR-0632T, 1994, and references therein.
  - [12] CDF: F. Abe et al., Phys. Rev. Lett. **73**, 225 (1994).
  - [13] A. Kusenko, unpublished.
  - [14] Recent results, some of which are preliminary, are from Ref. [15]. LEP results are from S. Bethke, talk presented at *Radiative Corrections: Status and Outlook*, Gatinsburg, TN, June 1994; and from Ref. [11]. Lattice results are from A. X. El Khadra et al., Ohio Report No. OHSTPY-HEP-T-93-020; C. T. H. Davies et al., OHSTPY-HEP-T-94-013. Other results are from I. Hinchliffe, in *Review of Particle Properties*, Phys. Rev. D **50**, 1297 (1994).
  - [15] B. R. Webber, Talk given at *The 27<sup>th</sup> International Conference on High Energy Physics*, Glasgow, Scotland, July 20-27, 1994; Cambridge Report No. CAVENDISH-HEP-94-15, 1994, and references therein.
  - [16] The  $\alpha_s$  discrepancy and its possible resolutions were discussed recently by M. Shifman, Minnesota Report No. UMN-TH-1323-94, 1994.
  - [17] L. Lavoura and L. Wolfenstein, Phys. Rev. D **48**, 264 (1993).
  - [18] U. Amaldi et al., Phys. Lett. B **281**, 374 (1992).
  - [19] P. Langacker and N. Polonsky, Phys. Rev. D **50**, 2199 (1994).
  - [20] N. Polonsky and A. Pomarol, Phys. Rev. Lett. **73**, 2292 (1994); Pennsylvania Report No. UPR-0627T, 1994.
  - [21] M. Carena, S. Pokorski and C. E. M. Wagner, Nucl. Phys. **B406**, 59 (1993).
  - [22] L. J. Hall, R. Rattazzi and U. Sarid, LBL Report No. 33997, 1993; M. Carena et al., Nucl. Phys. **B426**, 269 (1994).
  - [23] V. Barger, M. S. Berger, P. Ohmann, and R. J. Phillips, Phys. Lett. B **314**, 351 (1993).
  - [24] A. Faraggi and B. Grinstein, Nucl. Phys. **B422**, 3 (1994); B. Lynn, Stanford Report No. SU-ITP-93-22, 1993; P. Chankowski, Z. Pluciennik and S. Pokorski, Warsaw Report No. IFT-94-19, 1994; R. Barbieri, P. Ciafaloni and A. Strumia, Pisa Report No. IFUP-TH-47-94, 1994. After completion of this work we received a paper by J. Bagger, K. Matchev and D. Pierce, Johns Hopkins Report No. JHU-TIPAC-95001, 1995, where similar issues are discussed and large positive corrections to  $\alpha_s(M_Z)$  are also found.
  - [25] A. Sirlin, Phys. Lett. B **232**, 123 (1989).
  - [26] M. Drees, K. Hagiwara and A. Yamada, Phys. Rev. D **45**, 1725 (1992).
  - [27] J. Erler, P. Langacker and N. Polonsky, work in progress.
  - [28] A. Djouadi et al., Nucl. Phys. **B349**, 48 (1991); M. Boulware and D. Finnel, Phys. Rev. D **44**, 2054 (1991); G. Altarelli, R. Barbieri and F. Caravaglios, Phys. Lett. B **314**, 357 (1993); J. D. Wells, C. Kolda and G. L. Kane, *ibid.* **338**, 219 (1994); M. Carena and C. E. M. Wagner, CERN Report No. TH-7393-94, 1994; J. E. Kim and G. T. Park, Seoul Reports No. SNUTP-94-66; SNUTP-94-95, 1994; D. Garcia, R. A. Jimenez and J. Sola, Barcelona Report No. UAB-FT-343, 1994.
  - [29] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. R. Quinn and S. Weinberg, *ibid.* **33**, 451 (1974); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**,

- 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).
- [30] R. Arnowitt and P. Nath, Phys. Rev. Lett. **69**, 725 (1992); J. Hisano et al., Nucl. Phys. **B402**, 46 (1993); Tohoku Report No. TU-470, 1994.
  - [31] After completion of this work we received the papers by T. Dasgupta, P. Mamales and P. Nath, Northeastern Report, 1995; D. Ring, S. Urano and R. Arnowitt, Texas A&M Rept No. CTP-TAMU-01-95, 1995; where NRO's in the minimal  $SU(5)$  model are studied.
  - [32] We thank Savas Dimopoulos for stressing this point and for suggesting to plot Fig. 4.
  - [33] V. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988).
  - [34] See, for example, I. Antoniadis, J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **272**, 31 (1991); L. E. Ibanez, D. Lust and G. G. Ross, *ibid.* **272**, 251 (1991); A. Faraggi, *ibid.* **302**, 202 (1993); L. E. Ibanez, *ibid.* **318**, 73 (1993); T. Kobayashi, D. Snematsu and Y. Yamagishi, *ibid.* **329**, 27 (1994).